EVOLUTION OF TRANSIENT THERMAL PROCESSES IN LAYER COUNTERFLOW APPARATUSES AND HEAT EXCHANGERS

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Results are given of an analytic investigation of transient processes inside counterflow apparatuses and heat exchangers with temperature disturbance in one of the heat carriers at the entry to the apparatus.

Nonstationary heat processes can arise if the operation of a counterflow apparatus is disturbed by a temperature change at the entry to a heat carrier or its consumption in the apparatus. The evolution of these processes determines the character as well as the time at which the new stationary state is reached.

The mathematical description of transient states of continuously operating counterflow apparatuses and heat exchangers has been dealt with in a number of articles [1-3]. In them the main attention was focused on the calculations of transient heat processes and their characteristics in the output sections of the heat exchangers only.

In more general cases and in particular in the analysis of the dynamics of processes taking place inside the apparatuses it is indispensable that one should know the particular features of thermal phenomena as yet not steady and in intermediate sections to be able to determine the dynamic characteristics of apparatuses as control objects and to chose the positioning of transducers which control the thermal state. These particular features can only be established if the corresponding problems are solved.

The following problem is considered below: in a counterflow apparatus for heating dispersion material by means of a gas moving through a layer some distribution of temperatures occurs which can either be nul (at the instant of connecting the apparatus) or it can correspond to some stationary state (operation of the apparatus prior to the change of state). At the time instant $\tau = 0$ the change in temperature of one of the heat carriers occurs at the entry to the apparatus and from now on it remains constant. It is required to find temperature changes in time of each heat carrier for any apparatus cross-section.

The following simplifications have been adopted in the formulation of the problem.

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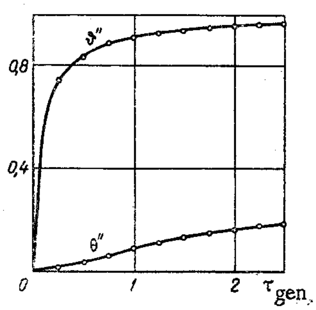

Fig.1. Temperature of heat carriers at the exit of a counterflow heat exchanger ($m = 0.8$; $Y_0 = 10$). ϑ'') material; θ'') gas; the points were obtained by analytic computations; the curves were found by using finite differences.

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TABLE 1. The Values of Imaginary Roots

M	n	σ_{1n}	σ_{2n}	σ_{3n}	σ_{4n}	σ_{5n}	σ_{6n}
0,5	0	—	—	—	—	—	—
	1	—	—	—	—	—	—
1,0	0	—	—	—	—	—	—
	1	—	—	—	—	—	—
2,0	0	—	—	—	—	—	—
	1	5,0371	—	—	—	—	—
4,0	0	—	—	—	—	—	—
	1	5,6162	—	—	—	—	—
6,0	0	3,8350	5,2260	—	—	—	—
	1	5,8204	—	—	—	—	—
8,0	0	3,6097	5,5215	—	—	—	—
	1	5,9275	10,6393	11,0989	—	—	—
10,0	0	3,4991	5,6792	—	—	—	—
	1	5,9939	10,2098	11,5648	—	—	—
14,0	0	3,3859	5,8520	10,2458	11,5909	—	—
	1	6,0723	9,8311	11,8913	—	—	—
20,0	0	3,3078	5,9796	9,9452	11,9274	16,6957	17,7570
	1	6,1331	9,7622	12,1018	16,4350	18,0113	—
30,0	0	3,2501	6,0791	9,7560	12,1494	16,2816	18,1978
	1	6,1817	9,6432	12,2576	16,1567	18,3191	22,7014
50,0	0	3,2058	6,1597	9,6184	12,3175	16,0344	18,4712
	1	6,2216	9,5534	12,3805	15,9674	18,5366	22,3862

1. Physical heat properties of the layer material or gas are independent of temperature.
2. The internal heat resistance in portions forming the layer is taken into account by the solidity coefficient m_1.

With the above taken into account one obtains a system of equations which describes a heat exchange: in the flow of the dispersion material one has

$$\frac{\partial\vartheta}{\partial Z}+m\frac{\partial\vartheta}{\partial Y}=\theta-\vartheta, \qquad (1)$$

and in the gas flow

$$\frac{\partial\theta}{\partial Y}=\theta-\vartheta. \qquad (2)$$

The solution of the system under the conditions

$$Y=0,\ \vartheta=0; \qquad (3)$$

$$Y=Y_0,\ \theta=1; \qquad (4)$$

$$Z=0,\ \vartheta=0, \qquad (5)$$

describes the process prior to the stationary state and also, as shown by B. N. Devyatov in [1], the transient process between one stationary state and the next one.

The solution of the problem is obtained by using the operational method. Having taken the Laplace transformation with respect to the variable Z and having solved the system thus obtained of ordinary differential equations for the transform one obtains the formulas

$$\bar{\vartheta}=\frac{1}{s}\cdot\frac{Y^0}{m}\exp\left[-\frac{m-1}{2m}(Y_0-Y)\right]\frac{\operatorname{sh}\left(\sqrt{\delta^2-M^2}\,\frac{Y}{Y_0}\right)\exp\left[\frac{s}{2m}(Y_0-Y)\right]}{\delta\operatorname{sh}\sqrt{\delta^2-M^2}+\sqrt{\delta^2-M^2}\operatorname{ch}\sqrt{\delta^2-M^2}}, \qquad (6)$$

$$\bar{\theta}=\frac{1}{s}\exp\left[-\frac{m-1}{2m}(Y_0-Y)\right]\frac{\delta\operatorname{sh}\left(\sqrt{\delta^2-M^2}\,\frac{Y}{Y_0}\right)+\sqrt{\delta^2-M^2}\operatorname{ch}\left(\sqrt{\delta^2-M^2}\,\frac{Y}{Y_0}\right)}{\delta\operatorname{sh}\sqrt{\delta^2-M^2}+\sqrt{\delta^2-M^2}\operatorname{ch}\sqrt{\delta^2-M^2}}\exp\left[\frac{s}{2m}(Y_0-Y)\right], \qquad (7)$$

for the transformed temperatures $\bar{\vartheta}$ and $\bar{\theta}$ where $\delta = (1 + m + s/2m)Y_0$ and $M = (Y_0/\sqrt{m})$. By setting the denominator equal to zero one obtains the characteristic equations for the roots: hence it follows that one of the roots is $s_0 = 0$ and the other roots can be found from the equation

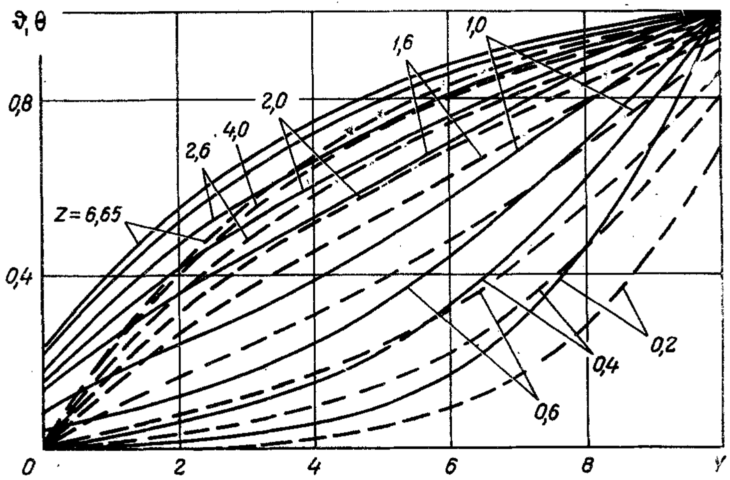

Fig.2. Temperatures of heat carriers along the length of the heat exchanger for various time instants (m = 0.8; Y_0 = 10). Continuous lines correspond to gas; dashed lines correspond to material.

$$\frac{\delta \operatorname{sh} \sqrt{\delta^2 - M^2} + \sqrt{\delta^2 - M^2} \operatorname{ch} \sqrt{\delta^2 - M^2}}{\sqrt{\delta^2 - M^2}} = 0. \tag{8}$$

For $\delta \to \pm M$ Eq. (8) reduces to

$$\delta + 1 = \pm M + 1 = 0. \tag{8a}$$

It can easily be seen that (8a) can hold if and only if $\delta = -M = -1$. Consequently, in the case of M = 1 Eq. (8) has one real root $\delta = -1$ that is $s = -(1 + \sqrt{m})^2 = -(1 + Y_0)^2$.

If, however, $\delta \neq -1$ the roots of Eq. (8) can be determined by setting the numerator equal to zero, that is,

$$\delta \operatorname{sh} \sqrt{\delta^2 - M^2} + \sqrt{\delta^2 - M^2} \operatorname{ch} \sqrt{\delta^2 - M^2} = 0. \tag{8b}$$

Following [2], Eq. (8b) is transformed and the notation $\delta = M \operatorname{ch} \Psi$ introduced. One obtains then

$$\operatorname{sh}(\Psi + M \operatorname{sh} \Psi) = 0. \tag{9}$$

The roots of Eq. (9) are $i\pi n$, where n = 0, 1, 2, 3, ... In this case one has

$$\Psi + M \operatorname{sh} \Psi = \pm i\pi n. \tag{10}$$

If the quantity Ψ is replaced by its value $\Psi = \mu \pm i\nu$ one obtains the system of equations

$$M \operatorname{sh} \mu \cos \nu = -\mu; \tag{11}$$

$$M \operatorname{ch} \mu \sin \nu = -\nu \pm \pi n. \tag{12}$$

It was established by further analysis that n can only assume a single even value and a single odd value, for example, 0 and 1. Other values of n do not produce any new roots which would be different from the case of n = 0 or 1.

It is noticed that the system (11)-(12) has an infinite number of roots. These equations, similarly as Eq. (10), have also purely imaginary roots ($\mu = 0$) $\nu = \sigma$ whose number is finite for $M \neq \infty$. In the case of $M \to \infty$ which corresponds to an infinitely large heating surface, the number of purely imaginary roots tends to infinity. The results for $M \to \infty$ were previously obtained in [4]. When M = 1 the system (19)-(20) of [4] has for n = 1 the imaginary root $\sigma = \pi$. It can be shown that this root corresponds to the value $\delta = -1$; thus, it is not a multiple root.

Using the information on the roots of the denominator of the expressions (6) and (7) one can determine the original functions ϑ and θ. After some transformations the following expressions are obtained which describe the transient processes between the first stationary state and the second one or transient processes related to the setting of the apparatus in motion (for $\delta \neq -1$):

TABLE 2. The Values of Complex Roots

M	n	ν_{1n}	μ_{1n}	ν_{2n}	μ_{2n}	ν_{3n}	μ_{3n}	ν_{4n}	μ_{4n}	ν_{5n}	μ_{5n}	ν_{6n}	μ_{6n}
0,5	0	4,0741	3,0078	10,6516	3,8119	17,0341	4,2515	23,3694	4,5563	29,6851	4,7897	35,9908	4,9790
	1	10,5650	3,4890	16,9938	4,0556	23,3467	4,4154	29,6705	4,6798	35,9805	4,8888	42,2828	5,0617
1,0	0	4,2124	2,2507	10,7125	3,1031	17,0734	3,5511	23,3984	3,8588	29,7081	4,0937	36,0099	4,2838
	1	10,6393	2,7687	17,0415	3,3522	23,3801	3,7168	29,6961	3,9831	36,0013	4,1933	42,3114	4,3668
2,0	0	4,3504	1,4875	10,7734	2,3937	17,1126	2,8503	23,4273	3,1612	29,7311	3,3976	36,0289	3,5885
	1	10,7234	2,0467	17,0893	2,6484	23,4135	3,0179	29,7218	3,2864	36,0221	3,4976	42,3171	3,6718
4,0	0	4,4720	0,5430	10,8317	1,6678	17,1510	2,1434	23,4559	2,4603	29,7538	2,6995	36,0478	2,8919
	1	10,8021	1,2885	17,1356	1,9351	23,4462	2,3147	29,7471	2,5871	36,0428	2,8003	42,3353	2,9757
6,0	0	10,8632	1,2104	17,1724	1,7187	23,4721	2,0446	29,7668	2,2876	36,0586	2,4820	42,3485	2,6443
	1	10,8412	0,7548	17,1611	1,5002	23,4647	1,8956	29,7615	2,1737	36,0546	2,3896	42,3454	2,5664
8,0	0	10,8838	0,8347	17,1869	1,4034	23,4832	1,7429	29,7758	1,9913	36,0662	2,1885	42,3550	2,3524
	1	17,1781	1,1682	23,4772	1,5888	29,7715	1,8752	36,0628	2,0949	42,3523	2,2739	48,6406	2,4251
10,0	0	10,8985	0,4325	17,1975	1,1415	23,4915	1,5012	29,7826	1,7571	36,0719	1,9579	42,3600	2,1239
	1	17,1904	0,8775	23,4865	1,3399	29,7789	1,6380	36,0690	1,8628	42,3576	2,0445	48,6452	2,1973
14,0	0	17,2124	0,6689	23,5033	1,1098	29,7924	1,3895	36,0802	1,6011	42,3672	1,7730	48,6535	1,9183
	1	17,2074	0,1205	23,4997	0,9229	29,7896	1,2611	36,0780	1,5015	42,3654	1,6910	48,6520	1,8484
20,0	0	23,5147	0,5866	29,8020	0,9548	36,0885	1,1968	42,3745	1,3837	48,6600	1,5377	54,9453	1,6694
	1	23,5122	0,1986	29,8000	0,7964	36,0869	1,0850	42,3731	1,2953	48,6589	1,4640	54,9684	1,6059
30,0	0	36,0971	0,6281	42,3821	0,8805	48,6669	1,0651	54,9515	1,2143	61,2359	1,3408	67,5203	1,4512
	1	36,0960	0,4415	42,3812	0,7667	48,6661	0,9784	54,9508	1,1431	61,2353	1,2799	67,5197	1,3977
50,0	0	54,9585	0,4422	61,2424	0,6589	67,5263	0,8048	73,8101	0,9410	80,0938	1,0486	86,3776	1,1432
	1	54,9581	0,2695	61,2421	0,5622	67,5260	0,7417	73,8098	0,8807	80,0936	0,9967	86,3773	1,0973

$$\vartheta = \frac{1 - e^{-\frac{1-m}{m}Y}}{1 - me^{-\frac{1-m}{m}Y_0}} + \sum_{n=0}^{1}\sum_{i=2}^{p}\frac{1}{\sqrt{m}}\exp\left\{-(Y_0 - Y)\left(1 - \frac{M}{Y_0}\cos\sigma_{in}\right)\right.$$

$$+\left[\frac{2m}{Y_0}M\cos\sigma_{in} - (1+m)\right]Z\Bigg\}\frac{(\sigma_{in} - \pi n)\sin\left[(\pi n - \sigma_{in})\frac{Y}{Y_0}\right]}{\cos\pi n\left[M\cos\sigma_{in} - \frac{Y_0}{2m}(1+m)\right](1+M\cos\sigma_{in})}$$

$$+2\sum_{n=0}^{1}\sum_{i=1}^{\infty}\frac{1}{\sqrt{m}}\exp\left\{-(Y_0 - Y)\left(1 + \frac{1}{Y_0}\mu_{in}\operatorname{cth}\mu_{in}\right) - \left[(1+m)\right.\right.$$

$$\left.\left.+\frac{2m}{Y_0}\mu_{in}\operatorname{cth}\mu_{in}\right]Z\right\}A_{in}^{-1}\left[(a_{2in}y_{1in} - a_{1in}y_{2in})\sin\left(Z\frac{2m}{Y_0}\mu_{in}\operatorname{tg}\nu_{in}\right) - (a_{1in}y_{1in} + a_{2in}y_{2in})\cos\left(Z\frac{2m}{Y_0}\mu_{in}\operatorname{tg}\nu_{in}\right)\right]; \quad (13)$$

$$\theta = \frac{1 - me^{-\frac{1-m}{m}Y}}{1 - me^{-\frac{1-m}{m}Y_0}} + \sum_{n=0}^{1}\sum_{i=1}^{p}\exp\left\{-(Y_0 - Y)\left(1 - \frac{M}{Y_0}\cos\sigma_{in}\right)\right.$$

$$+\left[\frac{2m}{Y_0}M\cos\sigma_{in} - (1+m)\right]Z\Bigg\}\frac{(\sigma_{in} - \pi n)\sin\left[\sigma_{in} - \frac{Y}{Y_0}(\sigma_{in} - \pi n)\right]}{\cos\pi n\left[M\cos\sigma_{in} - \frac{Y_0}{2m}(1+m)\right](1+M\cos\sigma_{in})}$$

$$-2\sum_{n=0}^{1}\sum_{i=1}^{\infty}\exp\left\{-(Y_0 - Y)\left(1 + \frac{1}{Y_0}\mu_{in}\operatorname{cth}\mu_{in}\right) - \left[(1+m) + \frac{2m}{Y_0}\right.\right.$$

$$\left.\left.\times\mu_{in}\operatorname{cth}\mu_{in}\right]Z\right\}A_{in}^{-1}\left[(a_{2in}x_{1in} - a_{1in}x_{2in})\sin\left(Z\frac{2m}{Y_0}\mu_{in}\operatorname{tg}\nu_{in}\right) - (a_{1in}x_{1in} + a_{2in}x_{2in})\cos\left(Z\frac{2m}{Y_0}\mu_{in}\operatorname{tg}\nu_{in}\right)\right]. \quad (14)$$

In the above

$$A_{in} = \cos\pi n\left\{\left[\mu_{in}\operatorname{cth}\mu_{in} + \frac{(1+m)Y_0}{2m}\right]^2 + (\mu_{in}\operatorname{tg}\nu_{in})^2\right\}[(1 - \mu_{in}\operatorname{cth}\mu_{in})^2 + (\mu_{in}\operatorname{tg}\nu_{in})^2];$$

$$y_{1in} = K[\mu_{in}B - (\nu_{in} - \pi n)C] + N[(\nu_{in} - \pi n)B + \mu_{in}C];$$

$$y_{2in} = K[(\nu_{in} - \pi n)B + \mu_{in}C] - N[\mu_{in}B - (\nu_{in} - \pi n)C];$$

$$a_{1in} = \left(\mu_{in}\operatorname{cth}\mu_{in} + Y_0\frac{1+m}{m}\right)(1 - \mu_{in}\operatorname{cth}\mu_{in}) + (\mu_{in}\operatorname{tg}\nu_{in})^2;$$

$$a_{2in} = \mu_{in}\operatorname{tg}\nu_{in}\left(1 - 2\mu_{in}\operatorname{cth}\mu_{in} - Y_0\frac{1+m}{m}\right);$$

$$x_{1in} = K[\mu_{in}D - (\nu_{in} - \pi n)E] + N[(\nu_{in} - \pi n)D + \mu_{in}E];$$

$$x_{2in} = K[(\nu_{in} - \pi n)D + \mu_{in}E] - N[\mu_{in}D - (\nu_{in} - \pi n)E];$$

$$K = \cos\left[\left(1 - \frac{Y}{Y_0}\right)\mu_{in}\operatorname{tg}\nu_{in}\right]; \quad N = \sin\left[\left(1 - \frac{Y}{Y_0}\right)\mu_{in}\operatorname{tg}\nu_{in}\right];$$

$$B = \operatorname{sh}\left(\mu_{in}\frac{Y}{Y_0}\right)\cos\left[(\nu_{in} - \pi n)\frac{Y}{Y_0}\right]; \quad C = \operatorname{ch}\left(\mu_{in}\frac{Y}{Y_0}\right)\sin\left[(\nu_{in} - \pi n)\frac{Y}{Y_0}\right];$$

$$D = \operatorname{sh}\left[\mu_{in}\left(1 - \frac{Y}{Y_0}\right)\right]\cos\left[\nu_{in} - (\nu_{in} - \pi n)\frac{Y}{Y_0}\right];$$

$$E = \operatorname{ch}\left[\mu_{in}\left(1 - \frac{Y}{Y_0}\right)\right]\sin\left[\nu_{in} - (\nu_{in} - \pi n)\frac{Y}{Y_0}\right].$$

It is not difficult to see that with $Z \to \infty$ the temperature distribution in the apparatus corresponds to a new stationary state. It is described by the first terms of the expressions (13) and (14) which are identical with the familiar formulas for counterflow heat exchangers [2].

For the case $\delta = -1$ one has to add to the above equations the terms

$$A_\vartheta = \frac{-6\frac{Y}{Y_0}}{(1+\sqrt{m})^2}\exp\left[-(1+\sqrt{m})\left(1 - \frac{Y}{Y_0}\right) - (1+\sqrt{m})^2 Z\right];$$

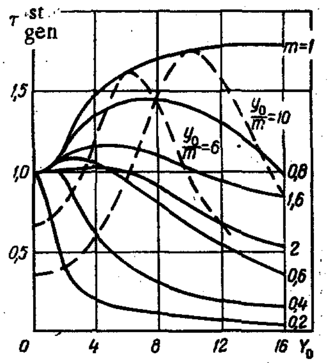

Fig. 3. Time needed for establishing the stationary state as dependent on the height and operation of heat exchangers. τ is the duration of the transient process. Continuous lines correspond to the parameter m = const; dashed lines correspond to the parameter Y_0/m = const.

$$A_\theta = \frac{-6\sqrt{m}\left(1-\frac{Y}{Y_0}\right)}{(1+\sqrt{m})^2}\exp\left[-(1+\sqrt{m})\left(1-\frac{Y}{Y_0}\right)-(1+\sqrt{m})^2 Z\right].$$

Numerical analysis has shown that the infinite series appearing in the expressions (13) and (14) converge rather slowly especially if the times under consideration are shorter than the time of one of the heat carriers remaining in the heat exchanger, that is, $Z = Y_0/m$. These cases are also of interest in practice since they are characteristic for heat exchangers either with small heat surface or with infinitely large heat surface. In both cases the time required for reaching the new stationary state is close to the time during which one of the heat carriers remains in the apparatus.

To facilitate the computations of the transient processes the values of the first six roots of Eqs. (10)-(12)* are shown in Tables 1 and 2 as dependent on M and n.

The comparison of the results of analytic and finite-difference computations was carried out for the heat carrier temperature at the exit of the apparatus. From now on the change in the temperature of the latter is adopted as a process stationarity criterion. Figure 1 shows the temperature of the heat carriers at the exit of the heat exchangers ϑ'', where the time during which the material stays in the apparatus was adopted as a time unit, that is, $\tau_{gen} = Z(m/Y_0)$. The quantity ϑ'' was calculated by two independent methods. These calculations have shown that in our formulation of the problem there are no vibrations in the transient process; the heat-carrier temperature at the exit from the heat exchanger varies monotonically tending to attain the level corresponding to the new stationary state. The analysis has established that to determine the transient process curve for $\tau_{gen} \geq 1$ it suffices to use 2-3 terms of the series. However, for $\tau_{gen} \ll 1$ the number of terms increases rapidly and for $\tau_{gen} = 0.25$ one must already use 10 terms of the series to reach a required accuracy. The required accuracy, therefore, can be obtained for $\tau_{gen} \geq 1$ by using the series and for $\tau_{gen} < 1$ one should use the formulas (22) and (23) of [4].

In Fig. 2 temperature distribution is shown of heat carriers for the case of m = 0.8 and $Y_0 = 10$ (M = 11.2) along the length of the heat exchanger at different time instants. It can be seen from the graph that the highest initial rate of change and the absolute magnitude of temperature change of the heating and heated media is observed at the exit of the apparatus (by the less heated heat carrier); as one approaches the entry to the heat exchanger (along these media) $\Delta\vartheta(\theta)$ and $d\vartheta(\theta)/dZ$ become smaller. It is also noticed that in practice the time needed for establishing the stationary state increases the further one is from the point of entry of the heating heat carrier whose temperature causes the disturbance.

One usually adopts the time required to reach $\vartheta'' = 0.95\vartheta''_{stat}$ as the stabilization time of the transient process. Computations of the latter are shown in Fig. 3. It can be seen from the graph that those heat exchangers whose water-equivalent ratio is close to unity have the longest stabilization time. For values of m greater or less than unity the duration of the transient process is reduced. For $m \to 0$ the duration of the transient process approaches zero: in this case the heating of the heat carrier takes place under constant temperature of the heating medium. For $m \to \infty$ the heat exchange stabilizes rapidly since the heat exchange is concentrated only at the entry portion to the heat exchanger.

*These calculations were carried out on a digital computer by the junior research worker M. V. Raeva.

NOTATION

$\vartheta = (t - t_0)/(T_0 - t_0)$, $\theta = (T - t_0)/(T_0 - t_0)$	are relative temperatures;
t, T	are the temperatures of material and gas respectively;
t_0, T_0	are the same for the initial state;
$Z = [\alpha_V m_1/c\rho(1 - w/w_g)][\tau - (y_0 - y)/w_g]$	is the dimensionless time;
$m_1 = 1/(1 + Bi/\nu)$	is the solidity coefficient;
$Bi = (\alpha_F R/\lambda)$	is the Biot number;
α_F, α_V	are the heat-exchange coefficients referred to 1 m^2 surface and 1 m^3 layer;
R	is the depth of heat penetration in a portion;
λ	is the portion heat conductivity coefficient;
ν	is the shape coefficient ($\nu = 0$ for a plate, $\nu = 1$ for a cylinder, $\nu = 2$ for a sphere);
c, c_g	are the heat capacities of material and gas respectively;
ρ, ρ_g	are the volumetric masses;
w, w_g	are the flow velocities of material and gas;
y	is the distance from the point of entry to the heating heat carrier;
y_0	is the heat-exchanger length;
$Y = \alpha_V m_1 y/w_g c_g \rho_g$	is the dimensionless coordinate;
$m = c\rho w/c_g \rho_g w_g$	is the water equivalent ratio.

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